The modeling of GRB spectra from poisson distributions to synchrotron emission

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Stop using the Band function, alpha, and any correlations derived from them for physics.
Very quick overview that leaves out most of the details but looks pretty.

**What can we learn?**

A large amount of energy is deposited in a small region. This is radiated in gamma-rays distributed as a Band function.

Said differently: we have no good idea what is producing the emission yet.
What is the origin of GRB prompt emission?

- The jet photosphere?
- Thermal-ish emission
- Shocks/reconnection?
- Non-thermal, optically-thin emission

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3 Myths

narrow spectra

the line-of-death

millisecond variability
What observations do we have?

- The raw count spectrum is indexed in channel energy and has units of electronic count per second.
- How can we use this to understand GRB emission physics?
What observations do we have?
What observations do we have?
What observations do we have?

Spectra are fit via a forward-folding analysis.

You get back what you put in.
What observations do we have?

Spectra are fit via a forward-folding analysis.

You get back what you put in.
What observations do we have?

Models that appear different in \( vF_v \) can be very similar in data space due to the effect of the response. This is why we must pay attention to the statistical procedures we use to fit data. For a beautiful example, see Vianello+ 2017.
The Multi-Mission Maximum Likelihood Framework

3ML

- point source
- data I
  - instrument I
  - plugin I
- data II
  - instrument II
  - plugin II
- data III
  - instrument III
  - plugin III

- diffuse source
- extended source
- neutrino source

- likelihood I
- likelihood II
- likelihood III

- parameter exploration
- plot
- model check
- propagate
- compare
- share
- save
- save
In the past (and currently), we want to use the low-energy spectral index of the Band function to infer physics.

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However, more recent studies have shown it is possible to fit synchrotron emission directly to count data.

Moreover, the predictions from photospheric models encompass a wide variety of alphas (Pe’er et al 2005 etc.).

We need another way to infer models from the data.
Define an auxiliary parameter from the Band function’s parameters that attempts to capture more information than alpha.

If one can distinguish between emission models via the width parameter, then we have a model comparison tool.

Axelsson & Borgonovo (2015)
Yu et al (2015)
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Axelsson & Borgonovo (2015)
Yu et al (2015)
The hypothesis is that thermal spectra are narrower and synchrotron spectra are very broad. Thus, if one can measure the width of the Band function, one can infer physics.
THE SPECTRAL WIDTH

Axelsson & Borgonovo (2015)
Yu+ (2015)
THE SPECTRAL WIDTH

Axelsson & Borgonovo (2015)
Yu+ (2015)
Synchrotron is once again strongly ruled out!

Axelsson & Borgonovo (2015)
Yu+ (2015)
Synchrotron fits to GRB data: too wide?

Burgess (2019)
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**Models that look very different in $vF_v$ space**

**can be very similar in count space.**
From data to physics

\[
\begin{align*}
\int_0^1 2x^2 \ln(x^2 + 1) \, dx &= \frac{1}{2} \ln(2) - \frac{1}{4} x^2 - x + \frac{x}{x^2 + 1} \\
&= \frac{1}{2} \ln(2) - \frac{1}{4} x^2 - \frac{1}{2} \int_0^1 \frac{2x}{x^2 + 1} \, dx \\
&= \frac{1}{2} \ln(2) + \left[ \frac{1}{4} \ln(2) - \frac{1}{2} \ln(x^2 + 1) \right]_0^1 \\
&= \frac{1}{2} \ln(2) + \frac{1}{4} \ln(2) - \frac{1}{2} \ln(2) + 0 = \frac{1}{4}.
\end{align*}
\]
Fit physical models directly to the data!
A history of synchrotron fits to GRB data
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Fokker-Planck equation

\[ \frac{\partial}{\partial t} n_e(\gamma, t) = \frac{\partial}{\partial \gamma} C(\gamma) n_e(\gamma, t) + Q(\gamma) \]

\[ Q(\gamma) \propto \gamma^{-p} \quad \forall \gamma \geq \gamma_{\text{inj}} \]

\[ C(\gamma) = -\frac{\sigma_T}{6\pi m_e c} B^2 \gamma^2 \]

power law injection \quad \text{synchrotron cooling}

synchrotron emission

\[ n_e(\epsilon; t) = \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} \epsilon n_e(\gamma, t) \Phi \left( \frac{2eB_{\text{crit}}}{3By^2} \right) \]

\[ \Phi(w) = w \int_{w}^{\infty} K_{5/3}(x) \, dx \]

\( \gamma_{\text{inj}} \) \quad \text{Injection electron energy} \quad p \quad \text{Injection spectral index}

\( \gamma_{\text{cool}} \) \quad \text{Cooling electron energy} \quad B \quad \text{Magnetic field strength}

Standard synchrotron emission model. No bells or whistles (or SSC/IC). The model allows us to test all synchrotron cooling regimes as a parameter!

The same number of parameters as the Band function

Burgess et al. 2019
The slope change is minimal until the distribution is extremely cooled. Thus, a lot of energy is extracted while still having a "slow-cooled" spectrum.

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How do we analyze the data?

The total counts are Poisson distributed

\[ \pi_{\mathcal{P}}(n \mid \lambda) = \frac{\lambda^n e^{-\lambda}}{n!} \]

The background is estimated from a model

\[ \pi_{\mathcal{BG}}(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{1}{2} \left( \frac{x - \mu}{\sigma} \right)^2} \]

This requires a joint PGstat distribution

\[ \pi_{\mathcal{PG}}(n_i \mid s_i, b_i, B_i, \sigma_{B_i}) = \pi_{\mathcal{P}}(n_i \mid s_i + b_i) \pi_{\mathcal{BG}}(B_i \mid b_i, \sigma_{B_i}) \]

Explore this likelihood with the appropriate priors

Any other likelihood is inappropriate and does not allow for further analysis such as model comparison!

\[ \mathcal{L} = \prod_{j=1}^{N_{\text{det}}} \prod_{i=1}^{N_{\text{chan}}} \pi_{\mathcal{PG}}(n_i^j \mid s_i, B_i^j, \sigma_{B_i^j}) \]

http://tinyurl.com/xrayspectra
We fit 18 single pulse GRBs with redshift.

168 time-resolved spectra

Burgess et al. 2019
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168 time-resolved spectra

Burgess et al. 2019
Posterior Predictive Checks

\[ \pi(\tilde{y} | y) = \int d\theta \, \pi(\tilde{y} | \theta) \pi(\theta | y) \]
PPCs express the volume in the posterior and the likelihood. Residuals only contain the information about the distance from data to model at one (non-unique) location on a surface.
We can compress the PPCs via QQ plots. However, this loses information.

Importantly, we can see that even when we simulate and fit the true model, fluctuations manifest.
A diverse population of cooling regimes.

However, most spectra are in the so-called “slow-cooling regime.”
The marginal distributions of alpha violate the line of death whilst the data are fully consistent with the data!

The band function is not a useful probe of physics!

Burgess et al. 2019
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The band function is not a useful probe of physics!

Burgess et al. 2019
The Band function is not a proxy for synchrotron!

Band function predicting narrower curvature of the data

Synchrotron also a good fit to the data
Imagine we have an object we want to image and model.
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Imagine we have an object we want to image and model. But our telescope doesn’t resolve the object well. The image is “dispersed.” But we find some empirical shape to fit the image data with. Now we compare our fitted model with some true physical model and find a disagreement.
We want to use our fits to compute physics about the outflows.

\[ \Gamma_{\text{obs}} \geq \Gamma_{\gamma\gamma} = \left( \frac{f_p L_{\text{obs}} \sigma_T}{r_0^2 m_e c^2} \right)^{1+2\beta} \]

A lower limit from pair-opacity

\[ \Gamma_{\text{obs}} \leq \Gamma_{\text{photosphere}} = \frac{\sqrt{2}}{4} \frac{e^{1/8} L_{\text{obs}}^{1/4} \sigma_T^{1/4}}{\pi^{1/4} r_0^{1/4} (1 + z)^{3/2} c^{3/4} m_p^{1/4}} \]

An upper limit from lack of a photosphere.

The peak of the emission is non-thermal in our model and we do not require an additional thermal component to model the data.

We conservatively assume that there could be a subdominant thermal component with 50% of the energy flux from the observed emission.
If we assume synchrotron emission, the fireball begins to smolder.
If we assume synchrotron emission, the fireball begins to smolder.
No fireball implies magnetization, but this pushes the emission site far beyond classical deceleration radius. Thus, we invoke comoving emission sites (some would say mini-jets) which allow for sane emission radii. See e.g. Beniamini et al (2018), Zhang & Yan (2011)
Millisecond Variability?

Golkhou et al. 2014-2015
Milliseccond variability is observationally rare... if existent at all.

This potentially allows for emission at larger radii and can be challenging for photospheric models.
Photospheric emission

Same GRB Different models

Ahlgren et al. 2019
The most recent attempts to fit sub-photospheric dissipation models have shown the model does not work.
Photospheric emission

Same GRB
Different models
GRB 160509A
Photospheric model does not work during main peak

Ahlgren et al. 2019

The most recent attempts to fit sub-photospheric dissipation models have shown the model does not work.
Ahlgren et al. 2019
This feature seems common in many bright GRBs
Polarization?
- Seen by GBM+POLAR
- Bayesian block temporal binning
- BALROG location consistent with IPN

Burgess, Kole et al. 2019
- Seen by GBM+POLAR
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GBM standard location
\[ \pi(\bar{p}, \phi, \psi | N) \propto \pi(\bar{p})\pi(\phi)\pi(\psi) \prod_k \pi(\lambda_k(\bar{p}, \phi; \psi) | n_k)\pi(\lambda_k^b | b_k) \prod_j \pi(\gamma_j(\psi) | m_j)\pi(\gamma_j^b | b_j) \]

Latent data:
\[ \lambda_k(\bar{p}, \phi; \psi) = \int d\varepsilon \, n_j(\varepsilon; \psi) R^k(\varepsilon, \phi, \bar{p}) \]
\[ \gamma_j(\psi) = \int_{\varepsilon_{1,j}}^{\varepsilon_{2,j}} d\varepsilon \, n_j(\varepsilon; \psi) R^j(\varepsilon) \]

Our full posterior relies on the **polarization** data and the **spectral** data.

The posterior **cannot be analytically** determined as proposed in the past.

This is why we have numeric integration routines!
POLAR+GBM data calibrated well

Emission described with synchrotron

No Band function!

See Merlin’s talk
- POLAR+GBM data calibrated well
- Emission described with *synchrotron*
- No Band function!
Future Work

• We will reanalyze the sample with various *photospheric* models including those of Asaf Pe’er, Andrei Beloborodov and Ramandeep Gill.

• Created a **fast**, parallel framework on an HPC that computes a grid of spectra. ~ **30K spectra every 5 days.** We can probably go faster!

• Models have been ported to a user friendly python interface.
Wrap up

- Fit the model you would like to learn about!
- Synchrotron is **alive** and well.
- Deeper comparison between models is needed.
- We do not need to spline away the universe.
- Alpha is not telling us much about physics… so neither are the correlations related to alpha!
- Independent results from Ravasio, Oganesyan, and Zhang
Wrap up

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- Synchrotron is alive and well.
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Backup Slides (stuff I had no time to cover)
Analysis with a Smoothly Broken Power Law

It is also possible to fit for the curvature directly in the data by fitting a smoothly broken power law (SBPL).

Therefore, curvature is found during the spectral deconvolution process allowing for a statistical measure of spectral curvature.

The alpha distribution shifts dramatically, and the curvature measure is much more consistent with synchrotron emission.

The SBPL changes its curvature naturally. The Band function has an inherent curvature.

Simulations show that the curvature measured with the SBPL is more consistent with synchrotron.
Empirical models are not hypotheses.

We can always design an empirical model that allows us to better predict the data. Thus, the goal of an empirical model is to predict the data without over-fitting.

How to we compare the models?

Information criteria: AIC, DIC, WAIC, LOO
Over interpretations

When empirical models go wrong

Simulating synchrotron spectra and then fitting with a Band function or a SBPL shows that these empirical models do a poor job of reproducing the spectral shapes.
Parameter Evolution
Electron Number

\[ R = 10^{14} \text{ cm} \]

\[ p(N_e) \]

\[ 10^{51} \quad 10^{52} \quad 10^{53} \quad 10^{54} \quad 10^{55} \quad 10^{56} \]

Graph showing distributions of electron number with GRB labels.
Spectral Index and B-Field

\[ R = 10^{14} \text{ cm} \]
Decaying B-Fields
Model(
  PointSource(name, ra, dec, spectrum, polarization)
  ExtendedSource(name, ra, dec, spectrum, shape)
  ParticleSource(name, ra, dec, spectrum)
  PointSource(name, ra, dec, spectrum)
  NeutrinoSource(name, ra, dec, spectrum)
)
Model(
  PointSource(name, ra, dec, spectrum, polarization)
  PointSource(name, ra, dec, spectrum)
  ExtendedSource(name, ra, dec, spectrum, shape)
  ParticleSource(name, ra, dec, spectrum)
  PointSource(name, ra, dec, spectrum)
  NeutrinoSource(name, ra, dec, spectrum)
)
Does the CR spectrum depend on the neutrino spectrum?

Fitting an extended source with an embedded point source?

*astromodels* supports arbitrary linking between parameters. This also allows for the specification of time-varying models.

```
PointSource(name, ra, dec, spectrum, polarization)  PointSource(name, ra, dec, spectrum)
ExtendedSource(name, ra, dec, spectrum, shape)     ParticleSource(name, ra, dec, spectrum)
PointSource(name, ra, dec, spectrum)               NeutrinoSource(name, ra, dec, spectrum)
```