Prompt and accurate sky localization of gravitational-wave sources

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Abstract

Accurate, precise and prompt sky localization of gravitational-wave sources is essential to the success of multi-messenger astronomy. One of the most accurate sky localizations we can obtain is from full signal parameter estimation obtained with the LIGO-Virgo LALInference software, done after a quick initial sky localization with the Bayestar software. While more accurate, the improved analysis can take on the order of months to complete for a binary neutron star event. To solve this issue, we develop a new technique to speed up the parameter estimation. Our technique speeds up the parameter estimation by a factor of $\mathcal{O}(10^4)$ and enables updating the sky localization within $\mathcal{O}(10)$ minutes after the detection.

Key words: gravitational waves — multi-messenger astronomy — parameter estimation

1. Importance of rapid parmeter estimation

On August 17, 2017, the LIGO-Virgo collaboration detected gravitational waves emitted by a binary neutron star (BNS) for the first time and designated it GW170817 (Abbott et al. 2017a). The associated γ-ray burst ~ 1.7 s after the merger time was also detected by the Fermi-GBM (Meegan et al. 2009). This association triggered extensive follow-up observations and the electromagnetic counterparts were detected in a broad band from X-ray to radio (Abbott et al. 2017b), which marks the dawn of multi-messenger astronomy.

One of the most important things for the success of the multi-messenger observations is prompt and accurate sky localization of gravitational-wave sources with gravitational-wave data. As soon as gravitational-wave signal is detected, the source is quickly localized by the Bayestar software (Singer et al. 2015), and the reconstructed location is sent out to the follow-up observers. After this initial localization, the LIGO-Virgo collaboration conducts more detailed Bayesian Monte-Carlo parameter estimation with the LALInference software (Veitch et al. 2015) and provides the updated localization. While it is more accurate, this improved analysis can take on the order of months to complete for a BNS event, which is too slow for the purpose of follow-up observations and increases the chance of the counterparts being missed. Therefore, it is very important to speed up the parameter estimation for the success of the follow-up observations.

2. Reduced Order Quadrature

Reduced Order Quadrature (ROQ) (Canizares et al. 2015; Smith et al. 2016) is a technique to speed up the parameter estimation used by the LIGO-Virgo collaboration. Here we briefly explain this technique.

In the parameter estimation, the following inner product is calculated millions of times,

$$(\boldsymbol{h}(\boldsymbol{\theta}), \boldsymbol{d}) \equiv 4\Re\left[\Delta f \sum_{l=1}^{L} \frac{\tilde{h}^*(f_l;\boldsymbol{\theta})\tilde{d}(f_l)}{S_{\mathrm{n}}(f_l)}\right], \tag{1}$$

where $\boldsymbol{h}(\boldsymbol{\theta})$ is the model waveform for the source parameters, $\boldsymbol{\theta}$, $\boldsymbol{d}$ is data, $S_{\mathrm{n}}(f)$ is the (one-sided) power spectral density of the detector's noise and L is the number of frequency bins. Since the signal from a BNS observed by a ground-based detector is long and high-frequency, it requires $L = \mathcal{O}(10^5)$ to accurately represent the signal, which means the inner product calculation requires $\mathcal{O}(10^5)$ times waveform calculations. ROQ is a technique to reduce these waveform calculations.

The basic idea is to approximate the model waveform by the linear combination of basis waveforms much fewer than $\mathcal{O}(10^5)$,

$$\boldsymbol{h}(\boldsymbol{\theta}) \simeq \sum_{k=1}^{K} c_k(\boldsymbol{\theta})\boldsymbol{e}_k. \tag{2}$$

Then the inner product can be reduced to

$$(h(\boldsymbol{\theta}), \boldsymbol{d}) \simeq \sum_{k=1}^{K} c_k^*(\boldsymbol{\theta})(\boldsymbol{e}_k, \boldsymbol{d}). \tag{3}$$

Since the inner product, $(\boldsymbol{e}_k, \boldsymbol{d})$, can be pre-computed before the parameter estimation, the calculation of the inner product can be reduced to the calculation of the coefficients, $c_k(\boldsymbol{\theta})$, whose number is K. Therefore, the computational cost is reduced by a factor of $\sim L/K$ and the parameter estimation is sped up by the same factor.

The previous works show it requires $K \sim \mathcal{O}(10^3)$ basis waveforms for the BNS signal, and the parameter estimation can be sped up by a factor of $\mathcal{O}(100)$. While this is significant improvement, the parameter estimation still takes from 6 hours to a day. Given that the optical radiation continues for a few days after the merger, it is still too slow. To solve this problem, we develop a new technique to speed up the parameter estimation further.

3. Restriction of parameter space based on trigger values

The idea of our technique is to tune the parameter-estimation follow-up with the trigger values reported by detection pipelines. The detection pipeline filters data with millions of template waveforms corresponding to various values of source parameters. Then, it reports a set of source parameter values maximizing the significance of the signal. With these trigger values, we can roughly guess the true values of the source parameters. Especially, we can restrict the source parameter space explored in the parameter estimation. This restriction significantly reduces the variations of the model waveforms and the number of basis waveforms required to approximate them, and hence speeds up the parameter estimation further.

3.1. The best measurable combinations

Our strategy is to rely on some combinations of source parameters and restrict their ranges before parameter estimation. Such combinations should be the best measurable combinations otherwise we can not restrict the parameter space and speed up the parameter estimation significantly.

The previous work, Ohme et al. (2013), studies such measurable combinations. We apply a similar method to the waveform incorporating up to the 1.5 Post-Newtonian (PN) phase contribution and got the following two combinations,

$$\mu^1 = 0.975\psi^0 + 0.207\psi^2 + 0.0833\psi^3, \tag{4}$$

$$\mu^2 = -0.220\psi^0 + 0.822\psi^2 + 0.526\psi^3. \tag{5}$$

ψ^0, ψ^2 and ψ^3 are the 0PN, 1PN and 1.5PN phase contributions given by

$$\psi^0 = \frac{3}{4}(8\pi\mathcal{M}f_{\mathrm{ref}})^{-\frac{5}{3}}, \tag{6}$$

$$\psi^2 = \frac{20}{9}\left(\frac{743}{336} + \frac{11}{4}\eta\right)\eta^{-\frac{2}{5}}(\pi\mathcal{M}f_{\mathrm{ref}})^{\frac{2}{3}}\psi^0, \tag{7}$$

$$\psi^3 = (4\beta - 16\pi)\eta^{-\frac{3}{5}}(\pi\mathcal{M}f_{\mathrm{ref}})\psi^0, \tag{8}$$

where

$$\mathcal{M} = \frac{(m_1 m_2)^{\frac{3}{5}}}{(m_1+m_2)^{\frac{1}{5}}}, \quad \eta = \frac{m_1 m_2}{(m_1+m_2)^2}, \tag{9}$$

$$\beta = \frac{1}{12}\sum_{k=1}^{2}\left[113\left(\frac{m_k}{M}\right)^2 + 75\eta\right]\chi_k, \tag{10}$$

m_1, m_2 and χ_1, χ_2 are the masses and the dimensionless spin components aligned to the orbital angular momentum respectively. Here, we apply $f_{\mathrm{ref}} = 200$ Hz and a representative power spectral density of the LIGO-Livingston detector in its second observing run (O2).

3.2. The ranges of μ^1 and μ^2

Next, we discuss how to decide the ranges of μ^1 and μ^2 given the trigger values. In this study, we consider up to the 2PN phase contributions, ψ^α $(\alpha = 0, 2, 3, 4)$.

First, we need to take into account the mismatch between the trigger values and the true values. Since the template bank used by a detection pipeline is a discrete set of template waveforms, the trigger values are slightly off from the true values in general. Assuming the loss of the signal-to-noise ratio (SNR) due to the mismatch is small, we obtain the following constraint,

$$\tilde{\Gamma}_{\alpha\beta}(\hat{\psi}^\alpha - \psi_{\mathrm{t}}^\alpha)(\hat{\psi}^\beta - \psi_{\mathrm{t}}^\beta) < 2, \tag{11}$$

where $\hat{\psi}^\alpha$ and ψ_{t}^α are the true value and the trigger value of ψ^α respectively. $\tilde{\Gamma}$ is the matrix obtained by projecting the constant time and phase out of the Fisher matrix (See the discussions around Eq. (17) of Ohme et al. (2013)).

We also need to take into account the statistical error due to the instrumental noise. In the limit of high SNR value, the posterior distribution can be approximated by a Gaussian distribution, and its N-sigma region is given by

$$\tilde{\Gamma}_{\alpha\beta}(\psi^\alpha - \hat{\psi}^\alpha)(\psi^\beta - \hat{\psi}^\beta) < \left(\frac{N}{\rho_{\rm net}}\right)^2, \qquad (12)$$

where $\rho_{\rm net}$ is the network SNR. The parameter space given by (12) becomes broader for a smaller value of $\rho_{\rm net}$, and we should apply the lowest possible value of $\rho_{\rm net}$. Since the detection usually requires the signal-to-noise ratio of $\gtrsim 8$, we apply conservative threshold, $\rho_{\rm net} = 5$.

We also note that the prior constraints on the masses and spins significantly affect the possible ranges of μ^1 and μ^2. Since we focus on BNS events, we consider the following prior range of the masses,

$$0\ M_\odot < m_1, m_2 < 3\ M_\odot. \qquad (13)$$

On the spins, we consider the following two different prior ranges,

$$\text{astrophysical-spin prior:}\ -0.05 < \chi_1, \chi_2 < 0.05. \qquad (14)$$

$$\text{high-spin prior:}\ -0.7 < \chi_1, \chi_2 < 0.7. \qquad (15)$$

The former one is motivated by the fact that even PSR J0737-3039A (Burgay et al. 2003), which is one of the observed binary neutron star system that will merge within a Hubble time and contains the most extremely spinning pulsar among them, will have $|\chi| \lesssim 0.04$ at the merger. The latter one is motivated by the fact that the maximum spin parameter of a uniformly rotating star is ~ 0.7 for various realistic nuclear equations of state (Lo et al. 2011).

The ranges of μ^1 and μ^2 are their possible ranges given the constraints (11), (12) and (14) or (15).

4. Performance

Finally, we constructed ROQ basis waveforms in the restricted parameter spaces and performed parameter estimation on the artificially injected BNS signals to see the performance of our technique.

4.1. ROQ basis waveforms

With the algorithm we developed in the previous section, we divided the whole parameter space into a lot of small $\mu^1 - \mu^2$ ranges. As a result, we found that we need 17007 and 7327 prior ranges for the astrophysical-spin and high-spin priors respectively with the detectors' sensitivities in the O2.

Next, we constructed ROQ basis waveforms of TaylorF2 waveform implemented in LALSuite (LIGO Scientific Collaboration 2018) starting from 20 Hz. Then we found that the number of the basis waveforms, K, is reduced to $27 - 40$ and $50 - 66$ for the astrophysical-spin and high-spin priors respectively. Therefore, with our technique, the parameter estimation can be sped up by a factor of $L/K = \mathcal{O}(10^4)$ and is faster than the conventional ROQ method by a factor of $\mathcal{O}(100)$.

4.2. Run time

We implemented μ^1 and μ^2 as sampling parameters to LALInference and performed the parameter estimation with our technique on a injected signal from a $1.4M_\odot$-$1.4M_\odot$ non-spinning binary neutron star. As a result, we found that the run times are reduced to ~ 10 minutes for the astrophysical-spin prior and ~ 20 minutes for the high-spin prior, which enables updating the sky localization of a BNS event within $\mathcal{O}(10)$ minutes after the detection.

5. Conclusion

The detailed Bayesian Monte-Carlo parameter estimation can provide the most accurate and precise sky localization of gravitational-wave sources. On the other hand, it can take months to complete for binary neutron star events and is too slow for their follow-up observations. To solve this issue, we developed a new technique to speed up the parameter estimation. The idea of our technique is to tune the parameter-estimation follow-up with the trigger values reported by detection pipelines. As a result, we sped up the parameter estimation by a factor of $\mathcal{O}(10^4)$, which enables providing the accurate and precise sky localization of a BNS event within $\mathcal{O}(10)$ minutes after its detection and increases the chance of the electromagnetic counterparts being detected.

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